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Dispersion Theoretical Analysis of Pion Photoproduction at Threshold and in the Delta Region

L. Tiator

Institut für Kernphysik, Universität Mainz, J.J. Becherweg 45

Abstract

A new partial wave analysis of pion photoproduction has been obtained in the framework of fixed-t dispersion relations valid form threshold up to 500 MeV. It is based on new Mainz data for π° and π^+ production off the proton and both older and more recent data from Bonn, Frascati and TRIUMF for π^+ and π^-. At threshold we obtain a good agreement with the existing data for both charged and neutral pion production. In the resonance region we have precisely determined the electromagnetic properties of the $\Delta(1232)$ resonance, in particular the E2/M1 ratio $R_{EM} = -2.5 \pm 0.1\%$. We show that a model independent resonance background subtraction is possible with the speed-plot technique and obtain the Δ pole at $W = (1211 - 50i)$ MeV and the E2/M1 ratio of the residues as $R_\Delta = -0.035 - 0.046i$.

INTRODUCTION

Dispersion theoretical analyses have been very successful in the description and understanding of pion nucleon scattering and pion photoproduction already in the 60s. Based on unitarity, analyticity, crossing symmetry, gauge invariance and Lorentz invariance they provide a powerful tool to investigate the low energy behaviour of the nucleon and the structure of nucleon resonances. During the last few years beams of high current and high duty factor together with considerably improved particle detection techniques have reduced the statistical errors to the order of a few percent, and promise to keep control of the systematical errors at the same level. To interpret these data with respect to the most interesting features, i.e. the threshold behaviour and the electromagnetic excitation of resonances, a partial wave analysis is mandatory. To ensure the consistency and uniqueness of such an analysis, constraints from unitarity and dispersion relations have to be imposed. Such concepts have proven to be quite successful in pion-nucleon scattering(1). In comparison with that field, the situation in pion photoproduction is considerably more complex. The spin and isospin structure leads to twelve independent amplitudes, while in pion-nucleon scattering there are only four such amplitudes. As a consequence a complete experiment requires the use of many polarization observables. Such a complete experiment has not yet been performed. However, the new experiments provide an ever increasing amount of precise and new data. At present, the experimental thrust is mainly on measurements near threshold and around the $\Delta(1232)$ resonance. In the coming years, a series of experiments at Jefferson Lab will cover the whole resonance region. Restricting our theoretical investigations to the threshold region and the low-lying resonances, we are lead to choose the method of Omnès and Muskhelishvili to analyse the existing data, because it introduces a natural parametrization and fulfills the constraints of unitarity at the same level.

DISPERSION RELATIONS AT FIXED t

Starting from fixed-t dispersion relations for the invariant amplitudes of pion photoproduction, the projection of the multipole amplitudes leads to a well known system of integral equations,

$$\mathrm{Re}\mathcal{M}_l(W) = \mathcal{M}_l^{\mathrm{P}}(W) + \frac{1}{\pi}\sum_{l'}\mathcal{P}\int_{W_{\mathrm{thr}}}^{\infty} K_{ll'}(W,W')\mathrm{Im}\mathcal{M}_{l'}(W')dW', \tag{1}$$

where $\mathcal{M}_l$ stands for any of the multipoles $E_{l\pm}, M_{l\pm}$, and $\mathcal{M}_l^{\mathrm{P}}$ for the corresponding (nucleon) pole term. The kernels $K_{ll'}$ are known, and the real and imaginary parts of the amplitudes

are related by unitarity. In the energy region below two-pion threshold, unitarity is expressed by the final state theorem of Watson,

$$\mathcal{M}^{\mathcal{I}}_{\updownarrow}(\mathcal{W}) =\mid \mathcal{M}^{\mathcal{I}}_{\updownarrow}(\mathcal{W}) \mid \rceil^{\rangle(\delta^{\mathcal{I}}_{\updownarrow}(\mathcal{W})+\backslash\pi)}, \tag{2}$$

where δ_l^I is the corresponding πN phase shift and n an integer. We have essentially followed the method of Schwela et al(2,3) to solve Eq. (1) with the constraint (2). In addition we have taken into account the coupling to some higher states neglected in that earlier reference. At the energies above two-pion threshold up to $W = 2$ GeV, Eq. (2) has been replaced by an ansatz based on unitarity(2). Finally, the contribution of the dispersive integrals from 2 GeV to infinity has been replaced by t-channel exchange, parametrized by certain fractions of ρ- and ω-exchange. Furthermore, we have to allow for the addition of solutions of the homogeneous equations to the coupled system of Eq. (1). The whole procedure introduces 9 free parameters, which have to be determined by a fit to the data.(4)

In our data base we have included the recent MAMI experiments for π° and π^+ production off the proton in the energy range from 160 MeV to 420 MeV(5,6,7), both older and more recent data from Bonn for π^+ production off the proton(8,9,10), and older Frascati(11) and more recent TRIUMF data(12) on π^- production off the neutron. Our fit obtained with this data base describes this data very well and in addition it also gives good agreement with data from the world data base (e.g. SAID(13)) not included in our fit.

RESULTS FOR THE THRESHOLD REGION

In Table 1 we give our results for the s- and p-wave amplitudes at threshold and compare them to the available information from heavy baryon chiral perturbation theory (HBChPT)(14) and experiment. The reduced p-wave amplitudes are defined as usual by $m_1 = \mathcal{M}_1/(\mid \vec{k} \mid\mid \vec{q} \mid)$, in the limit $k \to 0$. In fact the dependence on the photon energy q is not stringent, and in explicit calculations this definition leads to less energy dependence for e_{1+} and m_{1+}, while m_{1-} would vary less without the factor $\mid \vec{q} \mid$. In general there is a very good agreement between our analysis, HBChPT and experiment. In our analysis the uncertainties are mainly in the neutron channel, especially for $n(\gamma,\pi^0)n$, where the lack of precise experimental data, in particular of polarization observables, reflects in the threshold values. In fact it is interesting to note that the threshold amplitudes given in Table 1 are not fitted to the threshold data. They are only determined from experimental information above 160 MeV, therefore, they should be considered as predictions rather than fits. The interplay of the complete knowledge of pion photoproduction at all energies in the framework of dispersion relations can be seen in the individual contributions to the threshold values of the s-wave amplitudes. For $p(\gamma,\pi^0)p$ we obtain

$$E_{0+}^{\mathrm{thr}} = -7.63 + 4.15 + 2.32 - 0.41 + 0.29 + 0.07 = -1.22,$$

where the individual contributions are from the pole terms, from $M_{1+}, E_{0+}, E_{1+}, M_{1-}$ and higher multipoles, respectively.

There is special theoretical interest in the E_{0+} amplitude of $n(\gamma,\pi^-)p$ because it allows for an independent determination of the charge exchange pion-nucleon scattering length via the Panofsky ratio, $P = \sigma(\pi^-p \to \pi^0 n)/\sigma(\pi^-p \to \gamma n)$. This ratio is well determined by experiment, $P = 1.543 \pm 0.008$(15), and related to the scattering length by time reversal,

$$a_{CEX} \equiv a(\pi^-p \to \pi^0 n) = \sqrt{2\frac{q_0}{k_0}P}\; E_{0+}^{\mathrm{thr}}(\pi^-p), \tag{3}$$

where q_0 and k_0 are the cm momenta of photon and neutral pion at $p\pi^-$ threshold. Using our value of the threshold amplitude, and the measured Panofsky ratio, we find $a_{CEX} = (-0.120 \pm 0.002) \times m_\pi^{-1}$. This has to be compared with the value $(-0.129 \pm 0.002) \times m_\pi^{-1}$ resulting from

a partial wave analysis of pion-nucleon scattering(1) (solution KH80). Recently, a_{CEX} has also been determined by studying the level spacing of pionic atoms, with a preliminary value of $(-0.1301 \pm 0.0059) \times m_\pi^{-1}$(16).

Table 1. Threshold amplitudes for pion photoproduction. The s-waves E_{0+} are in units of $10^{-3}/m_{\pi^+}$ and the reduced p-wave multipoles are in units of $10^{-3}/m_{\pi^+}^3$. Our values are compared with results from chiral perturbation theory(14) and data analysis for charged pion production(17) and neutral pion production off the proton(5,18).

	$\gamma p \to \pi^+ n$				$\gamma n \to \pi^- p$			
	E_{0+}	m_{1-}	e_{1+}	m_{1+}	E_{0+}	m_{1-}	e_{1+}	m_{1+}
Disp.	28.0±0.2	6.1	4.9	-9.6	-31.7±0.2	-8.3	-4.9	11.2
ChPT	28.2±0.6				-32.7±0.6			
Exp.	28.3±0.3				-31.8±1.9			
	$\gamma p \to \pi^0 p$				$\gamma n \to \pi^0 n$			
	E_{0+}	m_{1-}	e_{1+}	m_{1+}	E_{0+}	m_{1-}	e_{1+}	m_{1+}
Disp.	-1.22±0.16	-3.92	-0.15	7.07	1.19±0.16	-2.16	-0.17	5.97
ChPT	-1.16	-3.21	-0.11	7.45	2.13	-1.63	-0.16	6.25
Exp.	-1.31±0.08	-3.38±0.26	-0.25±0.17	7.44±0.04				

RESULTS FOR THE RESONANCE REGION

According to Watson theorem, at least up to the two-pion threshold, the ratio $E_{1+}^{(3/2)}/M_{1+}^{(3/2)}$ is a real quantity. However, it is not a constant but even a rather strongly energy dependent function. If we determine the resonance position as the point, where the phase $\delta_{1+}^{(3/2)}(W = M_\Delta) = 90°$, we can define the so-called "full" ratio

$$R_{EM} = \left.\frac{E_{1+}^{(3/2)}}{M_{1+}^{(3/2)}}\right|_{W=M_\Delta} = \left.\frac{\mathrm{Im}E_{1+}^{(3/2)}}{\mathrm{Im}M_{1+}^{(3/2)}}\right|_{W=M_\Delta} . \tag{4}$$

We note that this ratio is identical to the ratio obtained with the K-matrix at the K-matrix pole $W = M_\Delta$. This can be seen by using the relation between the T- and the K-matrix, $T = K \cos\delta e^{i\delta}$ and consequently $K = \mathrm{Re}T + \mathrm{Im}T\, tan\delta$. Therefore, at $W = M_\Delta$ we find $K(E_{1+}^{(3/2)})/K(M_{1+}^{(3/2)}) = \mathrm{Im}E_{1+}^{(3/2)}/\mathrm{Im}M_{1+}^{(3/2)} = R_{EM}$. The recent, nearly model-independent value of the Mainz group at $W = M_\Delta = 1232$ MeV is $(-2.5 \pm 0.2 \pm 0.2)\%$(6) is in excellent agreement with our dispersion theoretical calculation that gives $(-2.5 \pm 0.1)\%$.

The analytic continuation of a resonant partial wave as function of energy into the second Riemann sheet should generally lead to a pole in the lower half-plane. A pronounced narrow peak reflects a time-delay in the scattering process due to the existence of an unstable excited state. This time-delay is related to the speed SP of the scattering amplitude T, defined by(19)

$$SP(W) = \left|\frac{dT(W)}{dW}\right| , \tag{5}$$

where W is the total $c.m.$ energy. In the vicinity of the resonance pole, the energy dependence of the full amplitude $T = T_B + T_R$ is determined by the resonance contribution,

$$T_R(W) = \frac{r\Gamma_R e^{i\phi}}{M_R - W - i\Gamma_R/2} , \tag{6}$$

while the background contribution T_B should be a smooth function of energy, ideally a constant. We note in particular that $W_R = M_R - i\Gamma_R/2$ indicates the position of the resonance pole in the complex plane, i.e. M_R and Γ_R are constants and differ from the energy-dependent widths, and possibly masses, derived from fitting certain resonance shapes to the data.

Applying this technique to our P_{33} amplitudes we find the pole at $W_R = M_R - i\Gamma_R/2 = (1211 - 50i)$ MeV in excellent agreement with the results obtained from πN scattering, $M_R = (1210 \pm 1)$ MeV and $\Gamma_R = 100$ MeV(19). The complex residues and the phases are obtained as $r_E = 1.23 \cdot 10^{-3}/m_\pi, \phi_E = -154.7^\circ, r_M = 21.16 \cdot 10^{-3}/m_\pi$ and $\phi_M = -27.5^\circ$, yielding a complex ratio of the residues

$$R_\Delta = \frac{r_E e^{i\phi_E}}{r_M e^{i\phi_M}} = -0.035 - 0.046i. \tag{7}$$

While the experimentally observed ratio R_{EM} is real and very sensitive to small changes in energy, the ratio R_Δ is a complex number defined by the residues at the pole, therefore, it does not depend on energy.

It should be noted, however, that a resonance without the accompanying background terms is unphysical, in the sense that only the sum of the two obeys unitarity. Furthermore we want to point out that the speed-plot technique does not give information about the strength parameters of a "bare" resonance, i.e. in the case where the coupling to the continuum is turned off. Both the pole position and the residues at the pole will change for such a hypothetical case, but the exact values for the "bare" resonance can only be determined by a model calculation and as such will depend on the ingredients of the model. In Table 2 we

Table 2. E/M ratios of different analyses. R_{EM} gives the "full" ratio $E_{1+}^{(3/2)}/M_{1+}^{(3/2)}$ at $W = M_\Delta$ and R_Δ gives the complex ratio obtained by the speed-plot technique at the resonance pole. All numbers are given in percentage.

analysis	R_{EM} [%] at $W_\Delta = 1232$ MeV	R_Δ [%] at $W_R = (1211 - 50i)$ MeV
VPI (SP97)(13)	-1.4	-3.1 -5.0 i
VPI (B500)(21)	-2.5	-4.0 -3.5 i
RPI(20)	-3.19	-4.8 -4.6 i
this work	-2.54	-3.5 -4.6 i
Mainz experiment(6)	$-2.5 \pm 0.2 \pm 0.2$	
BNL experiment(22)	$-3.0 \pm 0.3 \pm 0.2$	

compare our result with two VPI solutions, the solution of the RPI group in a field theoretical Lagrangian approach and the experimental analyses of Mainz and Brookhaven. With the numerical solutions of VPI and RPI we have applied the speed-plot technique in order to separate resonance and background contributions and to determine the pole position and residues. While the "full" ratios R_{EM} vary by more than a factor of 2 among these solutions, the ratios R_Δ are much closer to each others. In particular, the imaginary parts are very stable within only about 30%.

Finally, we have determined the photon couplings $A_{1/2}$ and $A_{3/2}$ of the delta resonance. From our energy-dependent analysis we get $A_{1/2} = (-132 \pm 2)$ and $A_{3/2} = (-253 \pm 3)$, both in units of $10^{-3}/\sqrt{GeV}$. However, it should be again noted that both the R_{EM} ratio and the photon couplings, calculated at the K-matrix pole are well-defined quantities but they have no direct connection to quark model calculations of a "bare" resonance.

SUMMARY

With the new and very precise data obtained at MAMI in Mainz we have obtained a new partial wave analysis for pion photoproduction. The uncertainties in most multipoles could be considerably improved compared to previous analyses. Very accurate results can be obtained at threshold and in the resonance region. At resonance we must clearly distinguish between the resonance position $W_\Delta = 1232$ MeV on the real axis and the pole at $W_R = (1211-50i)$ MeV in the complex plane. At the resonance position, where the phase passes 90°, we obtain an REM ratio of $R_{EM} = (-2.5 \pm 0.1)\%$ in very good agreement with the experimental analysis(6). This was also recently confirmed in a VPI analysis with a restricted data base(21). At the pole in the complex plane we obtain the ratio of the resonant electric and magnetic multipoles as $R_\Delta = -0.035 - 0.046i$. This is a model-independent ratio that can be determined in any analysis or calculation of pion photoproduction. After a long time of confusion about the different ratios that can be defined and constructed out of the measured cross sections or the analysed multipoles $E_{1+}^{(3/2)}$ and $M_{1+}^{(3/2)}$, it now appears that the ratio R_Δ is the closest one can get to a background subtracted value. Such a ratio must be complex, and it will be a challenge for all microscopic models to determine this ratio.

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